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MODELLING AND PREDICTING TURBULENCE FIELDS AND THE DISPERSION OF DISCRETE PARTICLES TRANSPORTED BY TURBULENT FLOWS

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Abstract—Turbulence fields are predicted with the aid of a (K-ϵ) model supplemented with algebraic relations deduced from a second-order closure scheme. Then, the dispersion of discrete particles transported by the turbulence fields predicted above is computed. Modelling of the discrete particle dispersion is based on an Eulerian approach. The (monodispersed) particles are considered as a continuous field for which a transport equation is written. The transport equation contains a dispersion tensor which is computed in the framework of the (slightly extended) Tchen theory, assuming a two-parameter family of Lagrangian correlation functions for the fluid particles. Modifications can be included to account for crossing-trajectory effects. Predictions are compared with experiments from Snyder & Lumley (1971), Wells (1982, 1984) and Arnason (1982).

1. INTRODUCTION

The knowledge and understanding of multiphase flows has increasingly caught the interest of researchers and engineers in recent years. In this paper, the discussion is limited to the simple case of turbulent two-phase flows where a dispersed phase is transported in the limit of a volume fraction tending to zero. The understanding and prediction of the dispersion of the discrete phase represent a thrilling challenge for the researcher who must face some fundamental and difficult problems such as, among a large collection of examples, the relation between Eulerian and Lagrangian spectral densities or the computation of particle turbulent transport coefficients. Furthermore, such a topic is essential for controlling and predicting certain processes and phenomena often closely connected with the present crisis of energy supply and also with care of the environment. Some examples are dispersive behavior of fuel droplets in energy conversion devices, or transport of particulate pollutants in the atmosphere, or in oceans, lakes, rivers.

The traditional approach for predicting the dispersion of discrete particles in turbulent flows is the so-called tracking method which uses a statistical simulation where the trajectories of the particles are computed with the aid of the equation of motion (Saffman 1965; Neilson & Gilchrist 1969; Domingos & Roziz 1974–1975; Base 1975, Swithenbank & Boyson 1978; among others). But the equations of motion are generally oversimplified. In a lot of cases, Newton's law is expressed with the drag force alone, given usually by the Stokes law. Furthermore this drag force is usually written as a function of the difference between the mean fluid and mean particle velocities, meaning that the basic equation used to describe the phenomenon is a purely deterministic one. The fundamental stochastic aspect of the turbulence is wiped out in such a model and a cloud of discrete particles with the same initial conditions does not disperse. Nevertheless, the phenomenon of particle dispersion through turbulent mechanisms can be reintroduced by a rather phenomenological attempt, using for instance, an equivalent random force leading to an extra-term in the particle momentum equation (Jurewicz & Stock 1976). The equivalent random particle force can be deduced from the turbulent kinetic energy K using a "diffusion approximation" or a "random force approximation" to use Dukowicz's terminology (1979). See also Lockwood *et al.* (1977); Abbas *et al.* (1980); and Crowe *et al.* (1977). But a fundamental analysis shows that the behavior of the particles does not really depend only on K but rather on all the harmonics of a Lagrangian spectral tensor. Furthermore, the possible introduction of dispersion through a dispersion term in the equations is linked to the problem of computing dispersion coefficients which are strongly dependent on particle inertia. This dependence could be deduced from experiments and empirically introduced in the model, but a better approach from a fundamental point of view would be to attempt theoretical predictions. The foregoing considerations have motivated the authors to develop an alternative to the tracking approach, evaluating the dispersion coefficients analytically from basic principles. The alternative is an Eulerian approach which is worthwhile in itself and also expected to provide valuable information for further advances of the tracking approach which is very far from being fully developed.

In the Eulerian approach, the discrete particles are considered as a continuous field $\bar{n}$, or a set of continuous fields, depending on whether the (spherical) particles are monosized, or not. Only monodispersed clouds will be discussed here. The continuous field $\bar{n}$ can be taken as the mean number of particles per unit volume. It complies with a transport equation in which the dispersion term contains a dispersion tensor depending on some fluid and turbulence characteristics, and on particle properties.

This paper reports on the computer program DISCO-2 (DISpersion COmputing) which aims at predicting turbulence fields and discrete particle dispersion in the framework of an Eulerian approach. The structure of the present work is basically the same as the structure of the associated code. The (K-ϵ) model and the supplemented algebraic relations for turbulent field predictions, the transport equation for $\bar{n}$, the dispersion tensor prediction, and some modifications required to account for some special effects such as the crossing trajectories, will be successively discussed. Finally, predictions from the code DISCO-2 will be compared with predictions from the code DISCO-1 (a previous, more costly and time-consuming version) and with experiments of Snyder & Lumley (1971), Wells (1982, 1984) and Arnason (1982).

2. TURBULENCE PREDICTIONS: THE (K-ϵ) MODEL

The first section of DISCO-2 is devoted to the turbulence predictions. Although assumptions concerning the particles will be later stated, we must here insist on the fact that this section strictly concerns the continuous phase in the absence of particles as our analysis is restricted to the case where the particle number-densities are small enough to avoid a significant modification of the fluid field by the discrete phase. This implies, for instance, that the viscosity μ presently used is a fluid molecular viscosity, not taking into account the presence of particles by a supplementary law like the classical Einstein one. Turbulence predictions are achieved by a (K-ϵ) model (present section) supplemented with algebraic relations from a second-order closure scheme (next section). The (K-ϵ) predictions are carried out with the aid of a parabolic version (boundary-later assumption in Spalding's terminology) (Patankar & Spalding 1972) of the TEACH-T computer program from the London Imperial College of Science and Technology (Gosman & Ideriah 1976). The notations are defined in figure 1. In the cylindrical (x, r, ϕ) coordinate system, the governing axisymmetric equations are, for steady flows,

- *Continuity equation:*

$$\frac{\partial}{\partial x}(r\rho U_x) + \frac{\partial}{\partial r}(r\rho U_r) = 0, \qquad [1]$$

where ρ is the fluid specific mass.

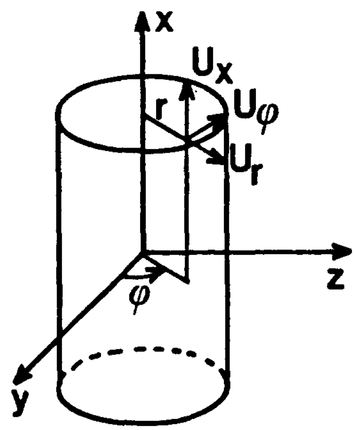

Figure 1. Cylindrical coordinates system.

- *Momentum equations:*

$$\frac{1}{r}\left(\frac{\partial}{\partial x}(r\rho U_x U_x) + \frac{\partial}{\partial r}(r\rho U_r U_x)\right) = -\frac{\partial P}{\partial x} + \frac{1}{r}\left[\frac{\partial}{\partial x}\left(r\mu_{\text{eff}}\frac{\partial U_x}{\partial x}\right) + \frac{\partial}{\partial r}\left(r\mu_{\text{eff}}\frac{\partial U_x}{\partial r}\right)\right] + S_{U_x}, \quad [2]$$

where P is the mean pressure.

$$\frac{1}{r}\left(\frac{\partial}{\partial x}(r\rho U_x U_r) + \frac{\partial}{\partial r}(r\rho U_r U_r)\right) = -\frac{\partial P}{\partial r} + \frac{1}{r}\left[\frac{\partial}{\partial x}\left(r\mu_{\text{eff}}\frac{\partial U_r}{\partial x}\right) + \frac{\partial}{\partial r}\left(r\mu_{\text{eff}}\frac{\partial U_r}{\partial r}\right)\right] - \mu_{\text{eff}}\frac{U_r}{r^2} + S_{U_r}. \quad [3]$$

Terms involving $\partial K/\partial x_i$ in the r.h.s. of [2] and [3] have been neglected since they produce only quite minor modifications in the results. Furthermore, we have

$$S_{U_x} = \frac{\partial}{\partial x}\left(\mu_{\text{eff}}\frac{\partial U_x}{\partial x}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu_{\text{eff}}\frac{\partial U_r}{\partial r}\right), \quad [4]$$

$$S_{U_r} = \frac{\partial}{\partial x}\left(\mu_{\text{eff}}\frac{\partial U_x}{\partial x}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu_{\text{eff}}\frac{\partial U_r}{\partial r}\right) - \mu_{\text{eff}}\frac{U_r}{r^2}. \quad [5]$$

The source terms S_{U_x} and S_{U_r} are additional terms corresponding to the spatial variation of the effective viscosity μ_{eff}. The effective viscosity is

$$\mu_{\text{eff}} = \mu + \mu_T, \quad [6]$$

where μ and μ_T are the molecular and turbulent viscosities, respectively.

- *Transport equation for* K, *the turbulence energy per unit of mass:*

$$\frac{1}{r}\left(\frac{\partial}{\partial x}(r\rho U_x K) + \frac{\partial}{\partial r}(r\rho U_r K)\right) = \frac{1}{r}\left[\frac{\partial}{\partial x}\left(r\frac{\mu_{\text{eff}}}{\sigma_K}\frac{\partial K}{\partial x}\right) + \frac{\partial}{\partial r}\left(r\frac{\mu_{\text{eff}}}{\sigma_K}\frac{\partial K}{\partial r}\right)\right] + S_K. \quad [7]$$

The source term S_K is

$$S_K = G - C_D\rho\epsilon, \quad [8]$$

where ϵ is the rate of energy dissipation and G is the production term which reads

$$G = \mu_T\left\{2\left[\left(\frac{\partial U_x}{\partial x}\right)^2 + \left(\frac{\partial U_r}{\partial r}\right)^2 + \left(\frac{U_r}{r}\right)^2\right] + \left(\frac{\partial U_x}{\partial r} + \frac{\partial U_r}{\partial x}\right)^2\right\} - \frac{2}{3}\mu_T\left(\frac{1}{r}\frac{\partial}{\partial r}(rU_r) + \frac{\partial U_x}{\partial x}\right)^2. \quad [9]$$

- *Transport equation for ϵ, the rate of dissipation of* K:

$$\frac{1}{r}\left(\frac{\partial}{\partial x}(r\rho U_x\epsilon) + \frac{\partial}{\partial r}(r\rho U_r\epsilon)\right) = \frac{1}{r}\left[\frac{\partial}{\partial x}\left(r\frac{\mu_{\text{eff}}}{\sigma_\epsilon}\frac{\partial\epsilon}{\partial x}\right) + \frac{\partial}{\partial r}\left(r\frac{\mu_{\text{eff}}}{\sigma_\epsilon}\frac{\partial\epsilon}{\partial r}\right)\right] + S_\epsilon, \quad [10]$$

where the source term S_ϵ is derived from a closure hypothesis:

$$S_\epsilon = C_1\frac{\epsilon G}{K} - C_2\rho\frac{\epsilon^2}{K}. \quad [11]$$

Finally, the turbulent viscosity μ_T is given by

$$\mu_T = C_\mu\rho\frac{K^2}{\epsilon}. \quad [12]$$

The closure constants are C_1, C_2, C_D, C_μ, σ_K and σ_ϵ, which are obtained empirically. We are only concerned here with parabolic flows leading to

$$\begin{aligned} \frac{\partial}{\partial x}\left(r\,\mu_{\text{eff}}\frac{\partial U_x}{\partial x}\right) &= \frac{\partial}{\partial x}\left(r\,\mu_{\text{eff}}\frac{\partial U_r}{\partial x}\right) = 0, \\ \frac{\partial}{\partial x}\left(r\frac{\mu_{\text{eff}}}{\sigma_K}\frac{\partial K}{\partial x}\right) &= \frac{\partial}{\partial x}\left(r\frac{\mu_{\text{eff}}}{\sigma_\epsilon}\frac{\partial\epsilon}{\partial x}\right) = 0. \end{aligned} \quad [13]$$

The computer program is then simplified by using a marching procedure, which means that computations are achieved, step by step, without requiring any grid sweeping. The equations are solved using a finite difference technique with standard boundary conditions as described in Gosman & Ideriah (1976).

The above set of equations can easily be specified for Cartesian coordinate systems by making $r \rightarrow y$, $(1/r)\,\partial/\partial_r \rightarrow \partial/\partial y$ and $U_r/r \rightarrow 0$.

The (K-ϵ) section thus predicts the mean velocities U_i, the turbulence energy per unit of mass K, the rate of energy dissipation ϵ, and the production of turbulence P, or G, depending on whether it is specified per unit of mass or per unit of volume. These quantities will be used as inputs in the second section devoted to algebraic relations deduced from a second-order closure scheme.

3. TURBULENCE PREDICTIONS: SECOND-ORDER ALGEBRAIC RELATIONS

The (K-ϵ) model predicts the turbulent energy K but does not give any precise information on its repartition according to the directions. It is necessary to know this here since we want to account for the nonisotropic character of the turbulence. The aim of this part is thus to predict the correlations of the fluctuating velocities. They can be computed from algebraic relations deduced from a second-order closure scheme. According to Rodi (1979) these algebraic relations read, in Cartesian coordinates,

$$\overline{u_i u_j} = K\left\{\frac{2}{3}\delta_{ij} + \frac{1}{(P - \epsilon + C_1'\epsilon)}\left[(1-\gamma_1)\left(P_{ij} - \frac{2}{3}\delta_{ij}P\right) - \gamma_2 K\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \gamma_3\left(D_{ij} - \frac{2}{3}\delta_{ij}P\right)\right]\right\}, \quad [14]$$

$$P_{ij} = -\left(\overline{u_j u_k}\frac{\partial U_i}{\partial x_k} + \overline{u_i u_k}\frac{\partial U_j}{\partial x_k}\right), \quad [15]$$

$$D_{ij} = -\left(\overline{u_j u_k}\frac{\partial U_k}{\partial x_i} + \overline{u_i u_k}\frac{\partial U_k}{\partial x_j}\right), \quad [16]$$

where u_i and U_i are the Eulerian fluctuating and mean velocity, respectively, in the ith direction, and C'_1, γ_1, γ_2, γ_3 are new closure constants. Note that P and K are scalars.

In the rigorous framework of the model, the constants γ_1, γ_2 and γ_3 are not independent but given by

$$\gamma_1 = \frac{(C_2 + 8)}{11}, \quad [17]$$

$$\gamma_2 = \frac{(30\,C_2 - 2)}{55}, \quad [18]$$

$$\gamma_3 = \frac{(8\,C_2 - 2)}{11}, \quad [19]$$

where the constant C_2 is usually taken as 0.4, leading to $\gamma_1 = 0.76$, $\gamma_2 = 0.18$ and $\gamma_3 = 0.11$. The set [14]–[16] can easily be specified for Cartesian 2D-flows, leading to a set of four equations with four unknown terms which are the four independent components of the symmetric tensor $\overline{u_i u_j}$ since all the other terms involved are constants or have been previously computed with the aid of the (K-ϵ) model. The set [14]–[16] can also be expressed for 2D-axisymmetric flows in the cylindrical coordinate system (x,r,ϕ), leading to (Berlemont & Gouesbet 1981; Gouesbet & Berlemont 1981)

$$P_{x\phi} = P_{r\phi} = D_{r\phi} = 0 \quad [20]$$

$$\overline{u_x u_\phi} = \overline{u_r u_\phi} = 0, \quad [21]$$

$$\overline{u_x^2} = K\left\{\frac{2}{3} + \frac{1}{P - \epsilon + C'_1\epsilon}\left[(1-\gamma_1)\left(P_{xx} - \frac{2}{3}P\right) - 2\gamma_2\frac{\partial U_x}{\partial x} - \gamma_3\left(D_{xx} - \frac{2}{3}P\right)\right]\right\}, \quad [22]$$

$$\overline{u_r^2} = K\left\{\frac{2}{3} + \frac{1}{P - \epsilon + C'_1\epsilon}\left[(1-\gamma_1)\left(P_{rr} - \frac{2}{3}P\right) - 2\gamma_2 K\frac{\partial U_r}{\partial r} - \gamma_3\left(D_{rr} - \frac{2}{3}P\right)\right]\right\}, \quad [23]$$

$$\overline{u_\phi^2} = K\left\{\frac{2}{3} + \frac{1}{P - \epsilon + C'_1\epsilon}\left[(1-\gamma_1)\left(P_{\phi\phi} - \frac{2}{3}P\right) - 2\gamma_2 K\frac{U_r}{r} - \gamma_3\left(D_{\phi\phi} - \frac{2}{3}P\right)\right]\right\}, \quad [24]$$

$$\overline{u_x u_r} = \frac{K}{P - \epsilon + C'_1\epsilon}\left[(1-\gamma_1)P_{xr} - \gamma_2 K\left(\frac{\partial U_x}{\partial r} + \frac{\partial U_r}{\partial x}\right) - \gamma_3 D_{xr}\right], \quad [25]$$

$$P_{xx} = -2\left(\overline{u_x^2}\frac{\partial U_x}{\partial x} + \overline{u_x u_r}\frac{\partial U_x}{\partial r}\right), \quad [26]$$

$$P_{rr} = -2\left(\overline{u_x u_r}\frac{\partial U_r}{\partial x} + \overline{u_r^2}\frac{\partial U_r}{\partial r}\right), \quad [27]$$

$$P_{\phi\phi} = -2\overline{u_\phi^2}\frac{U_r}{r}, \quad [28]$$

$$P_{xr} = -\left[\overline{u_x^2}\frac{\partial U_r}{\partial x} + \overline{u_x u_r}\left(\frac{\partial U_x}{\partial x} + \frac{\partial U_r}{\partial r}\right) + \overline{u_r^2}\frac{\partial U_x}{\partial r}\right], \quad [29]$$

$$D_{xx} = -2\left(\overline{u_x^2}\frac{\partial U_x}{\partial x} + \overline{u_x u_r}\frac{\partial U_r}{\partial x}\right), \quad [30]$$

$$D_{rr} = -2\left(\overline{u_x u_r}\frac{\partial U_x}{\partial r} + \overline{u_r^2}\frac{\partial U_r}{\partial r}\right), \quad [31]$$

$$D_{\phi\phi} = -2\overline{u_\phi^2}\frac{U_r}{r}, \quad [32]$$

$$D_{xr} = -\left[\overline{u_x^2}\frac{\partial U_x}{\partial r} + \overline{u_x u_r}\left(\frac{\partial U_x}{\partial x} + \frac{\partial U_r}{\partial r}\right) + \overline{u_r^2}\frac{\partial U_r}{\partial x}\right], \quad [33]$$

The corresponding system for 2D-Cartesian flows can be readily obtained from the above equations by making $x \to y$, $\phi \to z$, $(1/r)(\partial/\partial r) \to \partial/\partial y$ and $U_r/r \to 0$.

Two ways of solving the system [20]–[33] have been examined. The first way is to use a prediction-correction method (PCM). The basic idea is first to approximate the fluctuating velocity correlations with the aid of the (K-ϵ) gradient hypothesis equation, and then to correct them using the above second-order formalism. The precise procedure is as follows:

(i) Compute all the correlations $\overline{u_i u_j}$ with the aid of the (K-ϵ) gradient hypothesis:

$$\overline{u_i u_j} = -\frac{\mu_T}{\rho}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) + \frac{2}{3}\delta_{ij}K, \quad [34]$$

which becomes, for axisymmetric flows,

$$\overline{u_x^2} = -2\frac{\mu_T}{\rho}\frac{\partial U_x}{\partial x} + \frac{2}{3}K, \quad [35]$$

$$\overline{u_r^2} = -2\frac{\mu_T}{\rho}\frac{\partial U_r}{\partial r} + \frac{2}{3}K, \quad [36]$$

$$\overline{u_\phi^2} = -2\frac{\mu_T}{\rho}\frac{U_r}{r} + \frac{2}{3}K, \quad [37]$$

$$\overline{u_x u_r} = -\frac{\mu_T}{\rho}\left(\frac{\partial U_x}{\partial r} + \frac{\partial U_r}{\partial x}\right). \quad [38]$$

The tangential correlation is rather well predicted even at that stage. But the normal correlations depart significantly from experiments, and for a fully developed pipe flow (for instance), the tree normal correlations are found to be equal, an unacceptable result.

(ii) All the above approximate values are introduced in the r.h.s. of the set [20]–[33] to produce new values of the components $\overline{u_i u_j}$.

(iii) It would be possible to repeat the above process, namely by putting the new values in the r.h.s. of the equations to produce new estimates, and so on, thus carrying out an iterative procedure. Indeed, such a procedure does not converge in the cases we studied but gave birth to oscillatory diverging behavior for the tangential correlations. To avoid this problem, the iterations are carried out until convergence is obtained for the normal correlations, and after that, the tangential correlations are obtained by a single step. Actually, single step of this kind carried out on all the correlations has been found to give quite satisfactory results. Note that the above mentioned divergence of the whole iterative procedure is not an alarming feature since the results can be anyway obtained by the below discussed MPS having a well-defined and firm mathematical basis and leading to quite similar computed values.

(iv) When computing the tangential correlation, it has been found useful to include the hypothesis $P = \epsilon$ in the relation [25], leading to

$$\overline{u_x u_r} = \frac{K}{C_1' \epsilon}\left[(1 - \gamma_1)P_{xr} - \gamma_2 K\left(\frac{\partial U_x}{\partial r} + \frac{\partial U_r}{\partial x}\right) - \gamma_3 D_{xr}\right]. \qquad [39]$$

Such an assumption has been suggested by Ljuboja & Rodi (1979) and Hossain (1979). It produces an impressive improvement in the predictions, although it is rigorously true only in some special cases. In fact, P is rigorously equal to ϵ for a steady, homogeneous flow, where all mean quantities except U_i appearing in the transport equation for K do not depend on space, and where the mean strain rate tensor is constant (Tennekes & Lumley 1972).

The second way of solving the set is to recognize that it is actually a linear set of four equations with four unknowns ($\overline{u_x^2}$, $\overline{u_r^2}$, $\overline{u_\phi^2}$, $\overline{u_x u_r}$) and then to use standard procedures, such as a maximum pivot strategy (Carnahan *et al.* 1969). Results from the PCM have been found to be slightly better than from the MPS, with more simplicity in the program structure. Thus we finally decided to use only the PCM (although the MPS has a much better mathematical basis).

4. THE PARTICULATES TRANSPORT EQUATION

Turbulent quantities being predicted as described in sections 2 and 3, we have now to predict the behavior of discrete particles transported by the flow. Let $\bar{n}(x_i,t)$ be the number density of the (spherical and monosized) particles, the transport equation is written as

$$\frac{\partial \bar{n}}{\partial t} + U_i \frac{\partial \bar{n}}{\partial x_i} = \frac{\partial}{\partial x_i} \epsilon_{p,ij} \frac{\partial \bar{n}}{\partial x_j}, \qquad [40]$$

where we assume that the particles are convected with the fluid velocity U_i and where $\epsilon_{p,ij}$ is the turbulent dispersion tensor, the Brownian dispersion being neglected here. For steady and axisymmetric dispersion, that relation becomes in cylindrical coordinates

$$\frac{1}{r}\frac{\partial}{\partial x} rU_x\bar{n} + \frac{1}{r}\frac{\partial}{\partial r} rU_r\bar{n} = \frac{1}{r}\frac{\partial}{\partial x} r\epsilon_{p,xx}\frac{\partial \bar{n}}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r} r\epsilon_{p,rr}\frac{\partial \bar{n}}{\partial r} + \frac{1}{r}\frac{\partial}{\partial x} r\epsilon_{p,xr}\frac{\partial \bar{n}}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r} r\epsilon_{p,rx}\frac{\partial \bar{n}}{\partial x}. \qquad [41]$$

The dispersion tensor is predicted in the framework of the Tchen theory (1947) as explained later. Corrections can be introduced for special effects such as crossing-trajectories. A more general transport equation than [40] can be designed for the case where the particles are convected with $U_{p,i} \neq U_i$ but is not presented here.

5. PREDICTION OF THE TCHEN DISPERSION TENSOR

Only the necessary results will be recalled here as demonstration have been given elsewhere (Gouesbet & Berlemont 1979 a, b; Berlemont & Gouesbet 1982; Berlemont *et al.* 1983; Gouesbet *et al.* 1984). The dispersion tensor $\epsilon_{p,ij}$ reads

$$\epsilon_{p,ij} = \int_0^\infty E_{pL,(ij)}(\omega)\frac{\sin \omega t}{\omega} \mathrm{d}\omega, \qquad [42]$$

where $E_{pL,(ij)}$ is the symmetric part of the particulate Lagrangian spectral tensor $E_{pL,ij}$, ω the angular frequency corresponding to the frequency of a temporal harmonic oscillation into which the motion of a particle can be decomposed, and t the time of dispersion measured from the beginning of the dispersion process. $E_{pL,(ij)}$ can be computed from the symmetric part $E_{fL,(ij)}$ of the fluid Lagrangian spectral tensor $E_{fL,ij}$ through the simple relation

$$E_{pL,(ij)} = \eta^2 E_{fL,(ij)}, \quad [43]$$

where η is the Tchen amplitude ratio given by

$$\eta = [(1 + f_1)^2 + f_2^2]^{1/2}, \quad [44]$$

$$f_1 = \omega[\omega + c(\pi\omega/2)^{1/2}](b - 1)/D, \quad [45]$$

$$f_2 = \omega[a + c(\pi\omega/2)^{1/2}](b - 1)/D, \quad [46]$$

$$D = [a + c(\pi\omega/2)^{1/2}]^2 + [\omega + c(\pi\omega/2)^{1/2}]^2, \quad [47]$$

$$a = \frac{18\nu}{(s + 1/2)d^2}, \quad [48]$$

$$b = \frac{3}{2(s + 1/2)}, \quad [49]$$

$$c = \frac{9(\nu/\pi)^{1/2}}{(s + 1/2)d}, \quad [50]$$

where $s = \rho_p/\rho$ is the ratio of particle and fluid densities and ν the kinetic viscosity of the fluid.

The method of evaluating $E_{fL,(ij)}$ (or rather relevant components) will be explained later. We intend first to discuss the fact that $\epsilon_{p,ij}$ is generally a dispersion tensor involving nondiagonal components. Let us assume that we are able to compute the diagonal components of this tensor (we are in fact able to do this, a statement which will be examined later). Then, the process for computing the nondiagonal components is as follows. First note that $\epsilon_{p,ij}$,$E_{pL,(ij)}$,$E_{fL,(ij)}$ are symmetric tensors which can be made simultaneously diagonal by transforming the components from one Cartesian coordinate system x_k to another x'_k. But (Gouesbet et al. 1984),

$$\overline{u_{fL,i}u_{fL,j}} = \int_0^\infty E_{fL,(ij)}(\omega)\,d\omega. \quad [51]$$

Assuming that $\overline{u_{fL,i}u_{fL,j}} = \overline{u_i u_j}$ (which is rigorously true for homogeneous turbulence), the coordinate system x'_k, where $\overline{u_i u_j}$ is diagonal is the one where $E_{fL,(ij)}$ is diagonal. Knowing $\overline{u_i u_j}$ from the section 3 (second-order closure scheme), x'_k can be determined by a known rotation with respect to x_k. Then, in x'_k, the components $\epsilon'_{p,ij}$ (equal to zero for $i \neq j$) can be computed and, going back to x_k by the inverse transformation, the components $\epsilon_{p,ij}$ are obtained, including the nondiagonal components. Thus, the 3D-problem degenerates into a juxtaposition of three 1D-problems and, without any loss of generality, the formalism can be given in a monodimensional scheme. The mathematical expressions corresponding to the above process are not presented here since we shall not need them in the present work.

The transport equation [41] will be used here only for an axisymmetric parabolic flow of particles, namely the case of a particle point source in a fully developed turbulent pipe flow, for which the dispersion terms involving the derivatives $\partial/\partial x$ can be neglected, so that [41] becomes

$$\frac{1}{r}\frac{\partial}{\partial x} rU_x\bar{n} + \frac{1}{r}\frac{\partial}{\partial r} rU_r\bar{n} = \frac{1}{r}\frac{\partial}{\partial r} r\epsilon_{p,rr}\frac{\partial \bar{n}}{\partial r}. \qquad [52]$$

Then we only need to compute the diagonal component $\epsilon_{p,rr}$ which can be considered as a radial coefficient of dispersion $\epsilon_{p,r}$.

Note that nevertheless, predictions have been achieved both using [52] and the whole equation [41], including the nondiagonal components of the dispersion tensor, and have led to the same results, as expected.

Then, in a 1D-formalism, relations [42], [43] and [51] become

$$\epsilon_p = \int_0^\infty E_{pL}(\omega)\frac{\sin \omega t}{\omega}\,\mathrm{d}\omega, \qquad [53]$$

$$E_{pL}(\omega) = \eta^2 E_{fL}(\omega), \qquad [54]$$

$$\overline{u_{fL}^2} R_{fL}(\tau) = \int_0^\infty E_{fL}(\omega)\cos \omega\tau\,\mathrm{d}\omega, \qquad [55]$$

where the fluid Lagrangian correlation function $R_{fL}(\tau)$ reads

$$R_{fL}(\tau) = \frac{\overline{u_{fL}(0)u_{fL}(\tau)}}{\overline{u_{fL}^2}}. \qquad [56]$$

The problem is now to evaluate $E_{fL}(\omega)$. In a previous work which led to the so-called DISCO-1 code (Gouesbet *et al.* 1982; Berlemont & Gouesbet 1981), $E_{fL}(\omega)$ was evaluated by first building an Eulerian spectrum $E_{fE}(\omega)$ in the framework of the turbulence section then by applying an Euler–Lagrange simple rule of transformation. Although the results were found to be satisfactory for engineering purposes, that procedure was suffering from two main shortcomings: (i) computations of the dispersion coefficients required quadratures of oscillating functions which were costly and time-consuming, (ii) the Euler–Lagrange transformation was based on rather intuitive arguments. These shortcomings are avoided in the present work.

Here, we shall start from a fluid Lagrangian correlation function $R_{fL}(\tau)$ with two parameters, used by Frenkiel, and having empirical support (Frenkiel 1953; Calabrese & Middleman 1979; Snyder & Lumley 1971):

$$R_{fL}(\tau) = \exp\left(\frac{-\tau}{(m^2+1)\tau_L}\right)\cos\left(\frac{m\tau}{(m^2+1)\tau_L}\right), \quad \tau > 0, \qquad [57]$$

where τ_L is the Lagrangian time macroscale and the whole set of functions is generated when the real m increases from zero up. The parameter m is linked to the occurrence and the importance of negative loops in the function. By Fourier transforming, the spectrum $E_{fL}(\omega)$ is found to be

$$E_{fL}\omega) = \frac{2}{\pi}\overline{u_{fL}^2} f \frac{1+m^2+\omega^2 f^2}{(1-m^2+\omega^2 f^2)^2 + 4m^2}, \qquad [58]$$

where

$$f = (m^2+1)\tau_L. \qquad [59]$$

Thus, the coefficient of dispersion reads

$$\epsilon_p(t) = \int_0^\infty \eta^2(\omega)\,\frac{2}{\pi}\,\overline{u_{fL}^2}\,f\,\frac{1 + m^2 + \omega^2 f^2}{(1 - m^2 + \omega^2 f^2)^2 + 4m^2}\,\frac{\sin \omega t}{\omega}\,\mathrm{d}\omega. \qquad [60]$$

Generally, it again requires quadratures of oscillating functions. Nevertheless, very often, the so-called Basset term can be neglected in the equation of particle motion, leading mathematically to $c = 0$ (relation [50]). A discussion of the situations in which the Basset term can be neglected is available from Picart *et al.* (1982). In particular, such is the case in the situation we shall study later in this paper. Then, the quadrature in [60] can be performed analytically leading to

$$\epsilon_p = \overline{u_{fL}^2}\,\tau_L\{1 + \psi_0\,\mathrm{e}^{-at} + [\psi_1 \cos(mt/f) + \psi_2 \sin(mt/f)]\,\mathrm{e}^{-t/f}\}, \qquad [61]$$

where

$$\psi_0 = \frac{(m^2 + 1)(b^2 - 1)(1 + m^2 - a^2f^2)}{(a^2f^2 + m^2 - 1)^2 + 4m^2}, \qquad [62]$$

$$\psi_1 = \frac{-\{a^4f^4 - a^2f^2[(1 + b^2)(1 + m^2) - 4m^2] + b^2(1 + m^2)^2\}}{[a^2f^2 + m^2 - 1]^2 + 4m^2}, \qquad [63]$$

$$\psi_2 = m\psi_1 + 2m\,\frac{a^4f^4 - 2a^2f^2(1 - m^2) + b^2(1 + m^2)^2}{(a^2f^2 + m^2 - 1)^2 + 4m^2}. \qquad [64]$$

To close the problem, we have to evaluate $\overline{u_{fL}^2}$, m and τ_L. $\overline{u_{fL}^2}$ is approximated by $\overline{u^2}$ deduced from the second-order closure scheme section. The loop parameter m should be considered as a "closure constant" to be determined with the aid of experiments.

Note that, to avoid negative values for the dispersion coefficients, m must be smaller than 3.644 (Picart 1983) and a recommanded value is $m = 1$ (Calabresse & Middleman 1979; Gouesbet *et al.* 1984).

To evaluate τ_L, we know that ([19])

$$\mu_T \sim \rho\,\overline{u_{fL}^2}\,\tau_L, \qquad [65]$$

while, from the (K-ϵ) model (relation [12]),

$$\mu_T = C_\mu \rho\,\frac{K^2}{\epsilon}, \qquad [66]$$

where C_μ is about 0.09. This leads to

$$\tau_L \sim C_\mu\,\frac{K^2}{\epsilon}\,\frac{1}{\overline{\overline{u^2}}}, \qquad [67]$$

where we have estimated $\overline{u_{fL}^2}$ by $\overline{u^2}$.

At this point, there is one alternative. Noting, in isotropic flows, $K = \frac{3}{2}\overline{u^2}$, we could either eliminate K in [67] leading to

$$\tau_L \sim \frac{9}{4}\,C_\mu\,\frac{\overline{u^2}}{\epsilon} \sim 0.20\,\frac{\overline{u^2}}{\epsilon} \qquad [68]$$

or eliminate $\overline{u^2}$ in [67] leading to

$$\tau_L \sim \frac{3}{2} C_\mu \frac{K}{\epsilon} \sim 0.135 \frac{K}{\epsilon}. \qquad [69]$$

To choose between [68] and [69] we must point out that τ_L in [69] cannot be made dependent on the direction, while [68] can be rewritten in the ith direction as

$$\tau_{L,i} \sim 0.20 \frac{\overline{u_{(i)}^2}}{\epsilon} \qquad [70]$$

reintroducing the nonisotropic character of the flow. Note that τ_L is defined in 1D-problems as

$$\tau_L = \int_0^\infty R_{fL}(\tau)\, d\tau, \qquad [71]$$

which is a degenerate relation of the 3D-expression

$$\tau_{L,(ij)} = \int_0^\infty R_{fL,(ij)}(\tau)\, d\tau. \qquad [72]$$

Thus, τ_L owns a second-order tensorial character. But, in the coordinate system where $R_{fL,(ij)}$ is diagonal or when the nondiagonal components are not essential $\tau_{L,(ij)}$ will "look like" a vector $\tau_{L,i}$, justifying the choice of [70]. Finally, the constant 0.20 can be adjusted with the aid of experiments. Note, nevertheless, that this value has been previously used in predictions of scalar diffusion in a pipe flow (Dupont *et al.* 1984) and is expected to own some universal character.

6. MODIFICATIONS FOR CROSSING-TRAJECTORIES EFFECTS

As far as particulate dispersion is concerned, the Tchen theory is the only complete formal theory available at the present time. Thus, it could play the role of a limiting simple case, a reference framework for the discussion of current experiments and from which more general situations could be studied by introducing modifications in the basic theory.

One of the main assumptions in the Tchen theory is that a discrete particle must remain in the same fluid particle during its motion. Clearly, this cannot be true when there is a particulate drift produced by an external force field such as gravity or when particles are injected into the flow with a velocity which is not equal to U_i. Then, there is generally a significant lack of coincidence between the discrete particle and the fluid particle trajectories.

Yudine (1959) and Csanady (1963, 1973) considered the case of the turbulent diffusion of heavy particles in the atmosphere. These particles are supposed to fall quickly through the air, providing a sampling cut of the fluid particle velocities, so that $R_p(\tau)$ could be obtained from the fluid Eulerian space–time velocity autocorrelation in a frame of reference moving with the fluid mean velocity. Since Yudine the phenomenon of the lack of coincidence between particles and fluid particle trajectories is called the crossing-trajectory effect (CTE). Analyzing this phenomenon, Csanady derives an expression for the dispersion in the vertical direction, parallel to the free fall velocity:

$$\alpha_{p\|}(\infty) = \epsilon_{p\|}(\infty)\left[1 + \beta^2 \frac{\overline{f^2}}{\overline{\omega^2}}\right]^{-1/2}, \qquad [73]$$

where the coefficients of dispersion $\alpha_{p\parallel}$ and $\epsilon_{p\parallel}$ are discussed for an infinite time of dispersion (asymptotic values), $\epsilon_{p\parallel}$ is the coefficient of dispersion in the Tchen theory parallel to the free fall velocity f, β is the ratio τ_L/τ_E of Lagrangian and Eulerian time macroscales, respectively, having the same order of magnitude as 1, and $\overline{\omega^2}$ the mean square of the fluctuating velocities in the direction of the drifting force.

A similar formula is given for the dispersion in the horizontal direction (perpendicular to the free fall velocity). Nevertheless, because of the continuity of the flow, the formula is found to be slightly different:

$$\alpha_{p\perp}(\infty) = \epsilon_{p\perp}(\infty)\left[1 + 4\beta^2\frac{f^2}{\overline{\omega^2}}\right]^{-1/2}, \qquad [74]$$

where again only asymptotic values are considered and $\epsilon_{p\perp}$ is the coefficient of dispersion in the Tchen theory in the direction perpendicular to the free fall velocity. But these formulae cannot be in general rigorously valid, as, for instance, when the drift is not constant of when it is not large enough to provide an Eulerian sampling out as assumed in the analysis. A more complicated analysis has been given by Reeks (1977, 1980) but it was found to be too complex to be reasonably useful in a modelling for engineers. So, rather than complicate the scheme, the idea will be to simplify it and to obtain a semiempirical formula which will be then tested against experiments.

Let us focus our attention on a 1D-analysis in which the coefficient of dispersion α_p (a diagonal component of the dispersion tensor $\alpha_{p,ij}$ involved in the transport equation [47], including the effects due to $U_{p,i} \neq U_i$), writes

$$\alpha_p = f(\epsilon_p, \text{other variables}). \qquad [75]$$

The other variables are expected to be the turbulence energy K and the square of the mean velocity difference $(U_{p,i} - U_i)^2$. A dimensional analysis then leads to the nondimensional expression

$$\frac{\alpha_p}{\epsilon_p} = f\left[\frac{(U_{p,i} - U_i)^2}{K}\right]. \qquad [76]$$

To guess the form of the function f, we come back to Csanady's analysis and simply suggest

$$\alpha_p = \epsilon_p\left(1 + C_\beta\frac{(U_{p,i} - U_{f,i})^2}{2K/3}\right)^{-1/2}, \qquad [77]$$

where C_β is expected to have the same order of magnitude as 1. For particles falling freely and if the gravity is the only cause of $U_{p,i} \neq U_i$, [77] becomes (for asymptotic values)

$$\alpha_p = \epsilon_p\left(1 + C_\beta\frac{f^2}{2K/3}\right)^{-1/2}, \qquad [78]$$

which is to be compared with [73] and [74]. The simplification is mainly due to the fact that we no longer distinguish between $\alpha_{p\parallel}$ and $\alpha_{p\perp}$ and that C_β is an adjustable parameter (certainly nearly equal to 1) whose value will be deduced from experiments. Relation [77] will be found to be quite successful.

7. COMPARISONS BETWEEN THE CODES DISCO-1 AND DISCO-2

The turbulent field is the one reported by Laufer in 1954, namely a turbulent flow of air in a pipe with a diameter D equal to 0.25 m. The mean velocity on the axis is $U_M = 36$ m/s.

The Reynolds number is around 500,000, high enough to expect a good behavior of the (K-ϵ) model (which is better when the Reynolds number increases). The turbulent quantities, including the Reynolds tensor components, have been predicted using the model described in sections 2 and 3. Results, given elsewhere (Berlemont & Gouesbet 1981; Gouesbet & Berlemont 1981), are not repeated here. Comparisons between the predictions and the experiments have been found to be very satisfactory.

As far as coefficients of dispersion are concerned, the comparisons between the two codes are now given for particles transported in the above predicted turbulent field. Figure 2 shows the asymptotic longitudinal coefficients of dispersion $\epsilon_{p,x}(\infty)$ vs r/R, where r is the distance from the axis and R the radius of the pipe. Similarly, the asymptotic radial coefficients of dispersion $\epsilon_{p,ir}(\infty)$ are given in figure 3. The diameter and the nature of the particles do not matter for asymptotic values. Comparisons involve DISCO-1 results (previously published), then DISCO-2 results using either relation [69] or [70]. The agreement between DISCO-1 and DISCO-2 using relation [70] is very satisfactory. Relation [69] leads to less satisfactory results as expected from our discussion at the end of section 4 and will be from now on definitely abandoned.

Figures 4 and 5 show the ratios $\epsilon_{p,i}(t)/\epsilon_{p,i}(\infty)$ ($i = x$ or r) versus the time of dispersion at a distance $r = D/100 = 0.25$ cm from the axis. So near to the axis, the flow is locally isotropic and no difference is detected between the longitudinal and radial dispersion coefficients. The particles are water droplets having a diameter equal to 1 μm (figure 4) or 100 μm (figure 5). DISCO-1 is compared with DISCO-2. The agreement is fairly satisfactory (figure 4) or satisfactory (figure 5). Figures 6 to 9 show the ratios $\epsilon_{p,x}(t)/\epsilon_{p,x}(\infty)$ and $\epsilon_{p,r}(t)/\epsilon_{p,r}(\infty)$ for $r = 3.77$ cm. The nonisotropic character of the flow leads now to different values for the longitudinal and the radial coefficients. Again the comparisons are fairly satisfactory ($d = 1$ μm) or satisfactory ($d = 100$ μm). In all the cases, the recommanded value $m = 1$ has been used. Agreement is better when m decreases below 1 and worse when m increases.

The conclusion is that both codes DISCO-1 and DISCO-2 provide quite similar results. This is a first validation of these computer programs derived from our approach. In particular, it shows that the Euler–Lagrange rule of transformation used in DISCO-1 was useful at least for engineering purposes, as were our methods of predicting approximate Eulerian and Lagrangian spectra. Nevertheless, we prefer the code DISCO-2 which avoids the previously mentioned shortcomings of DISCO-1.

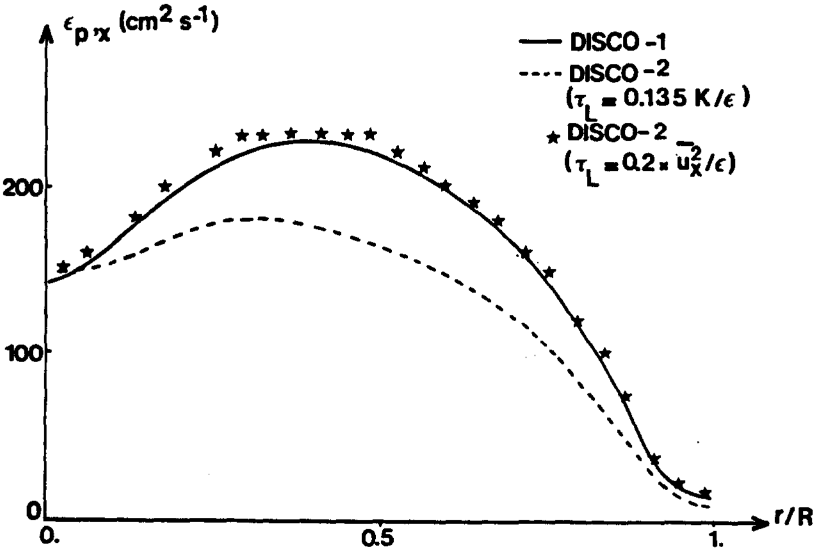

Figure 2. Radial dispersion coefficient: comparison between DISCO-1 and DISCO-2.

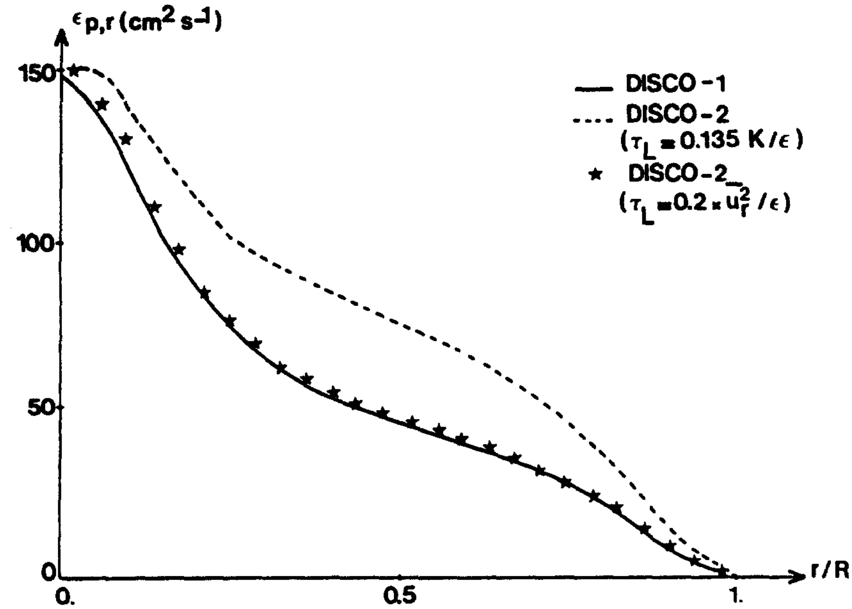

Figure 3. Longitudinal dispersion coefficient: comparison between DISCO-1 and DISCO-2.

8. COMPARISONS WITH SNYDER AND LUMLEY'S EXPERIMENTS

Snyder & Lumley (1971) studied the dispersion of various particles (hollow glass, solid glass, corn and copper) in a well-known turbulence, namely a stationary grid flow, nearly isotropic. The main direction of the air flow was vertical to avoid gravitational vertical drift of the particles with respect to the mean horizontal streamlines which would be encountered for a horizontal windtunnel. The section of the flow is a square of 40 × 40 cm^2 and the length of the test section is 5 m. In this situation, the particles remain reasonably far away from the walls whose influences can be neglected. The mean flow velocity is U = 6.5 m/s and the

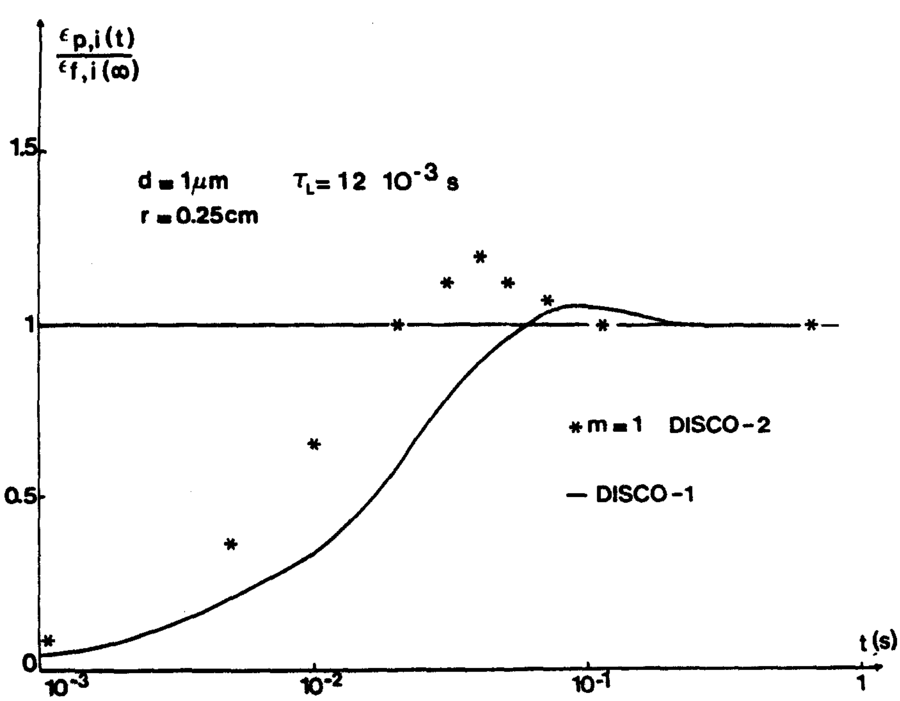

Figure 4. Dispersion coefficients in a pipe flow versus the time (d = 1 μm).

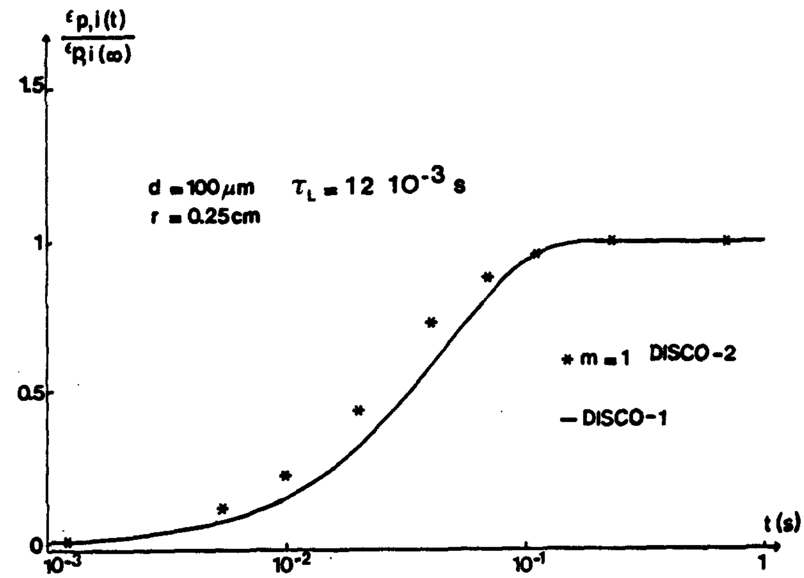

Figure 5. Dispersion coefficients in a pipe flow versus the time ($d = 100\ \mu$m).

Reynolds number based on the mesh dimension of the grid ($M = 2.54$ cm) is 10,000. Turbulence measurements were made and energy decay laws on the centerline of the test section established. The longitudinal and transversal variances $\overline{u_x^2}$ and $\overline{u_y^2}$ (equal to $\overline{u_z^2}$) respectively are given by

$$\frac{U^2}{\overline{u_x^2}} = 42.4\left(\frac{x}{M} - 16\right), \qquad [79]$$

$$\frac{U^2}{\overline{u_y^2}} = 39.4\left(\frac{x}{M} - 12\right), \qquad [80]$$

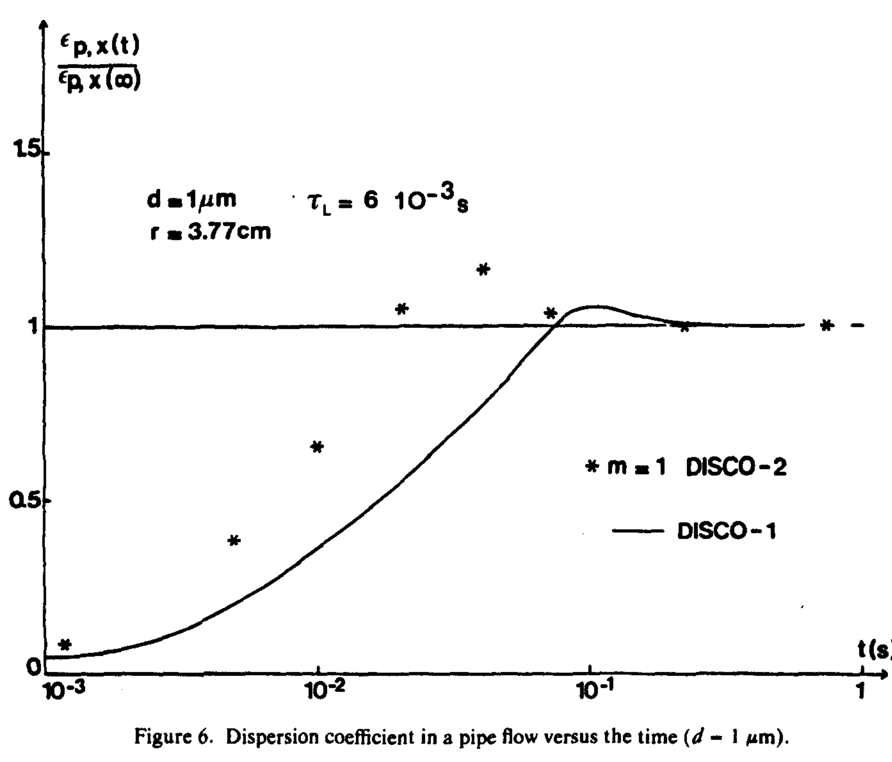

Figure 6. Dispersion coefficient in a pipe flow versus the time ($d = 1\ \mu$m).

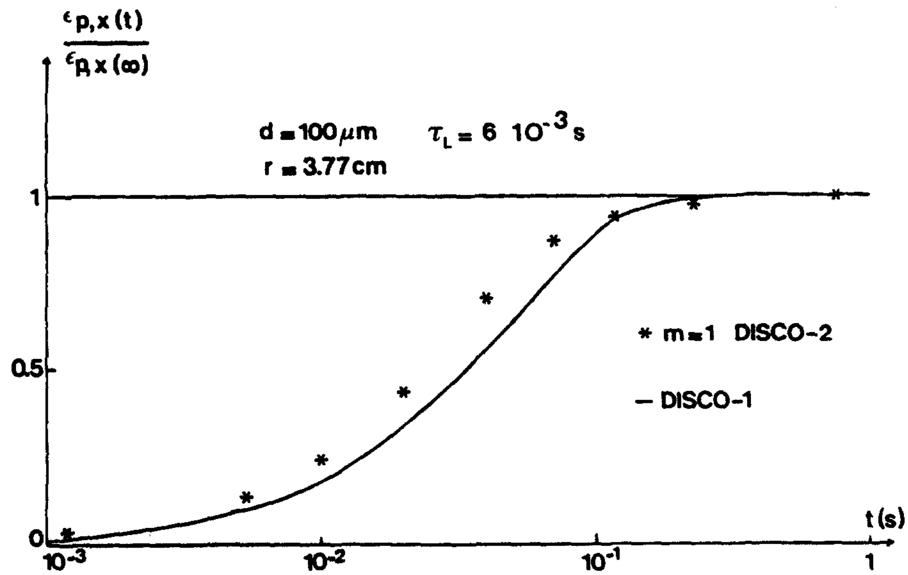

Figure 7. Dispersion coefficient in a pipe flow versus the time (d = 100 μm).

where x is the distance from the grid. The turbulence energy K is obtained from [79] and [80]. Then the rate of dissipation ϵ is given by

$$\epsilon = -U\frac{\mathrm{d}K}{\mathrm{d}x}. \tag{81}$$

ϵ has also been determined from the relation

$$\epsilon = 15\nu \int_0^\infty k^2 F_1(k)\,\mathrm{d}k, \tag{82}$$

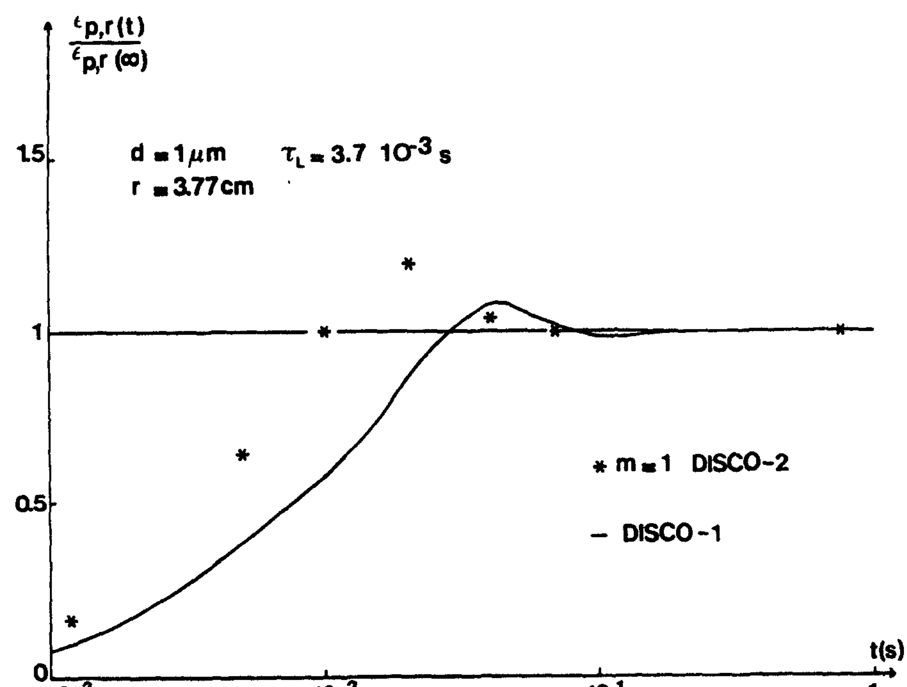

Figure 8. Dispersion coefficient in a pipe flow versus the time (d = 1 μm).

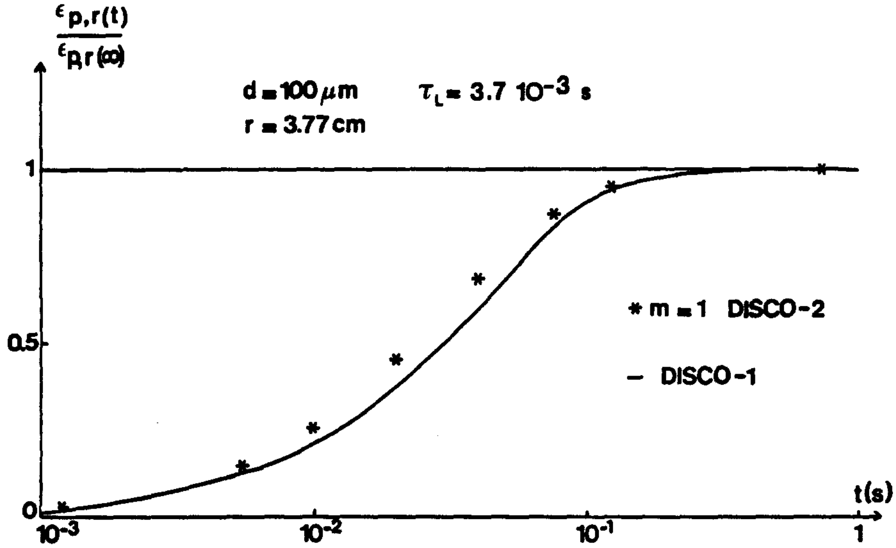

Figure 9. Dispersion coefficient in a pipe flow versus the time ($d = 100\ \mu$m).

where $F_1(k)$ is the spectral density of energy of the longitudinal component of the fluctuating velocity. A good agreement between results obtained from either [81] or [82] has been found. All the above results are sufficient for computing the transversal coefficient of dispersion $\epsilon_{p,y}$ in our approach or $\alpha_{p,y}$ when the corrections due to the crossing-trajectory effects are included. They will indeed be used for that purpose; the turbulence parts of DISCO-2 are not used in the present section.

The particle characteristics are given in table 1, where d and ρ_p have already been defined and where f is the terminal velocity, i.e. the Stokes velocity corrected by a method recommended by Fuchs (1964).

The particles are injected on the centerline of the flow, 51 cm after the grid with a mean velocity equal to the mean velocity of the flow. Measurements of the mean square displacements $\overline{y^2}$ were made at different locations, from 173 cm up to 427 cm from the grid. These experimental results can be compared with theoretical results, either $\overline{y_t^2} = 2\int_0^t \epsilon_{p,y}\,dt$ or $\overline{y_{t\mathrm{CTE}}^2} = 2\int_0^t \alpha_{p,y}\,dt$, depending on whether the crossing trajectory effects (CTE) are taken into account or not.

In figure 10, we compare $\overline{y^2}$ measured by Snyder & Lumley (points, stars . . .) and $\overline{y_t^2}$ computed from DISCO-2. Note that the results for solid glass and copper are the same (which is merely a coincidence). A very good agreement is obtained for the hollow glass particles which follow the fluid fairly well ($\tau_p/\tau_L \simeq 0.1$, where $\tau_p = 1/a$) and are not affected by the CTE, since their terminal velocity is small compared to the turbulence energy [$f = 1.67$, $2K/3 \sim 200$, so that $f^2(2K/3)^{-1} \sim 0.01$]. On the other hand, a large difference can be observed for the other particles which are strongly influenced by the CTE.

In figure 11 we compare $\overline{y^2}$ and $\overline{y_{t\mathrm{CTE}}^2}$ including the CTE-corrections. The agreement is very satisfactory. Note that C_β was taken equal to 0.85 (~ 1 as expected).

Table 1. Particle properties in Snyder & Lumley's experiments

	Hollow glass	Solid glass	Corn	Copper
d (μm)	46.5	87.0	87.0	46.5
ρ_p (g/cm^3)	0.26	2.5	1.0	8.9
f (cm/s)	1.67	44.2	19.8	48.3

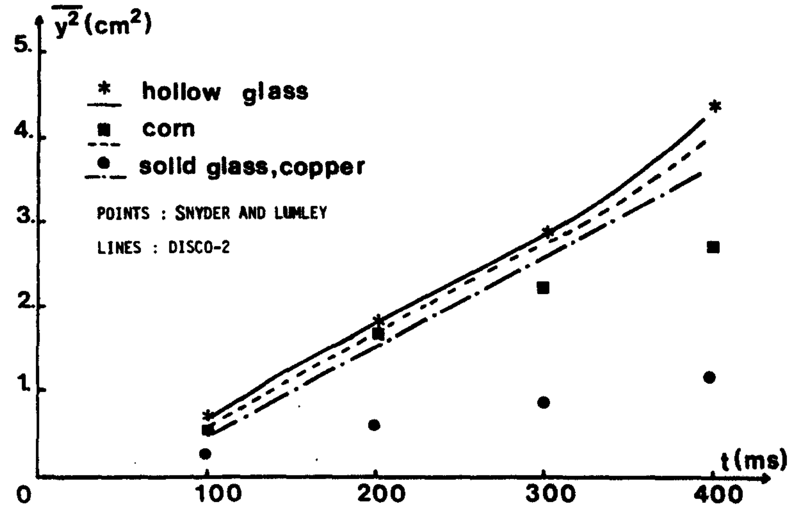

Figure 10. Comparison between DISCO-2 and Snyder & Lumley's experiment without the crossing-trajectories effects.

9. COMPARISONS WITH WELLS' EXPERIMENTS

The situation recently examined by Wells (1982) is essentially the same as the one examined by Snyder & Lumley, but the main direction of the flow is horizontal. Turbulence measurements were made and energy decay laws characterizing the grid flow are given, they are quite similar to Snyder & Lumley's results. The particles are glass spheres, with a diameter of 5 or 57 μm and a density equal to 2.45 g/cm^3. They are injected on the centerline of the flow and submitted to an electric field which allows the control of the crossing trajectory effects by adjusting the terminal velocity. Measurements of the mean square displacement $\overline{y^2}$ are made at different locations, from 50 cm up to 178 cm.

We introduced the decay laws for the turbulence given by Wells into the program, as we had previously done for Snyder & Lumley's results, and two cases are considered, depending on whether the correcting term for the CTE is used or not.

Figure 12 compares $\overline{y^2}$ vs x/M ($M = 2.54$ cm) measured by Wells (for $d = 5$ μm and a terminal velocity equal to 0) and $\overline{y^2} = \overline{y^2_{tCTE}}$ obtained from DISCO-2. There is no influence of the CTE effects (either in experiments or theory) and the agreement between experiments and predictions is very satisfactory.

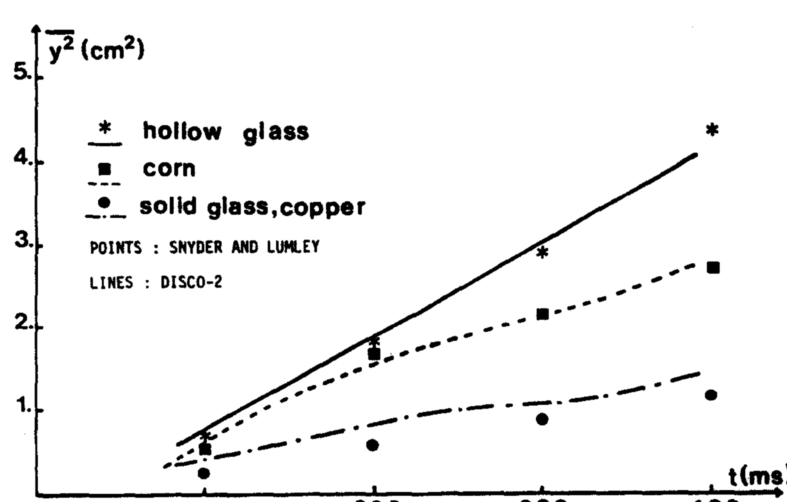

Figure 11. Comparison between DISCO-2 and Snyder & Lumley's experiment with the crossing-trajectories effects.

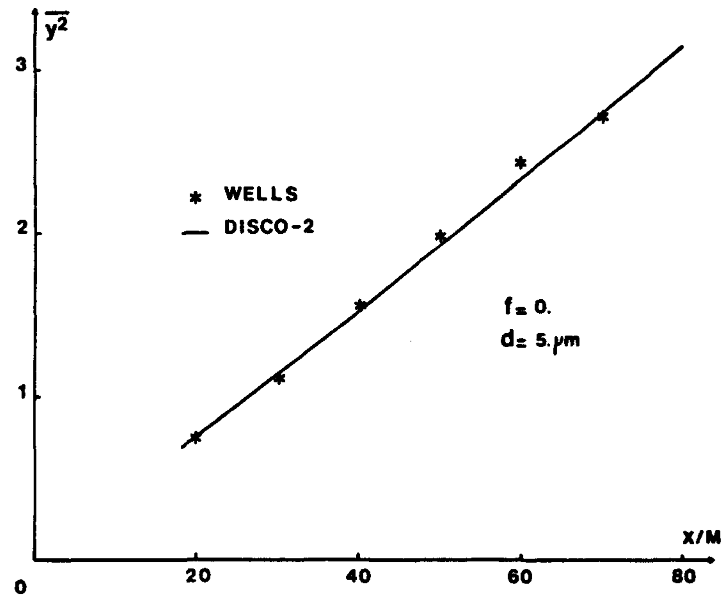

Figure 12. Comparison between DISCO-2 and Well's experiment.

Figure 13 compares the results for particles having a diameter equal to 57 μm and for four different terminal velocities, namely 0, 25.8, 39.7 and 54.4 cm/s. The CTE-corrections are included in the predictions (although they are not useful for $f = 0$). Again, the agreement is remarkable. Furthermore, note that C_β was taken to be equal to 0.85 as in the case of Snyder & Lumley's experiments. In particular, this shows that the relation [78] must be preferred (as stated) when compared with Csanady's relation [73] and [74], since the main flow was horizontal (perpendicular to the terminal velocity), while it was vertical (parallel to the terminal velocity) in Snyder & Lumley's situation.

10. COMPARISONS WITH ARNASON'S EXPERIMENTS

Arnason's experiments (1982) consist of an air flow in a pipe, aligned vertically to prevent gravity from changing the axial symmetry of the whole situation. The diameter of the pipe is 9 cm and the Reynolds number 50,000.

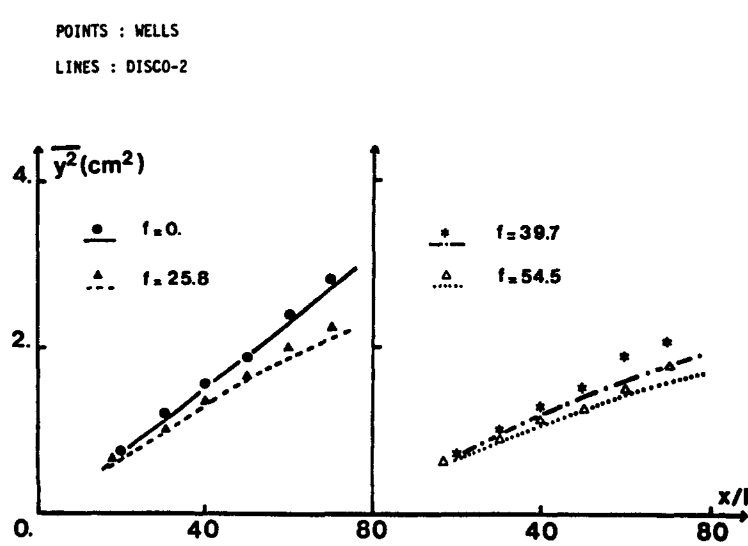

Figure 13. Comparison between DISCO-2 and Well's experiment.

The particles are the same as those used by Wells (1982) and are continuously and isokinetically injected on the centerline of the flow, 50 diameters after the beginning of the pipe in order to work in the fully developed part of the flow.

The whole code DISCO-2 has been used to predict the experimental situation, both for turbulence field and particle dispersion predictions, including the resolution of the transport equation [52]. The constants which have been used are

$$C_\mu = 0.09,\ C_1 = 1.56,\ C_2 = 1.92,\ C_D = 1.0,\ \sigma_\epsilon = 1.3,\ \sigma_K = 1,\ C_1' = 1.80,\ \gamma_1 = 0.76,$$
$$\gamma_2 = 0.18,\ \gamma_3 = 0.20.$$

The reader can observe that we used $\gamma_3 = 0.20$ instead of $\gamma_3 = 0.11$. This means that this constant has been adjusted separately. From a strict mathematical point of view, it is not a correct procedure because γ_1, γ_2, γ_3 must be linked together due to tensorial symmetry conditions. Nevertheless, from a practical point of view, this difference in γ_3 mainly affects the tangential correlation $\overline{u_x u_r}$ within only 15%. The idea in artificially improving the fitting between experiments and theory is to avoid the repercussion of turbulence prediction errors on the particle dispersion predictions. Nevertheless, even with $\gamma_3 = 0.11$, agreement remains satisfactory. Furthermore, the presented results concern only the normal correlations which are not affected by the modification of γ_3 from 0.11 to 0.20.

Figure 14 shows the mean velocity reduced profile U/U_M, where U_M is the centerline velocity versus $(1 - r/R)$. Predictions are compared with Arnason's and Laufer's results and the agreement is satisfactory.

Figures 15 and 16 show the fluid variances $\overline{u_x^2}$ and $\overline{u_r^2}$ respectively versus $(1 - r/R)$, compared with Arnason's and Laufer's measurements. Again, agreement is satisfactory.

Figures 17, 18 and 19 show the mean concentration reduced profiles C/C_M, where C_M is the concentration of particles on the axis, versus r/R for three different locations from the seeding point, namely 0.318, 0.502 and 0.679 m. The diameter of the particles is equal to 5 μm. The agreement is again very satisfactory, the slight discrepancies which appear on the edges of the profiles, being probably mainly due to the proximity of the walls.

11. GENERAL DISCUSSION

Before going to the conclusion, it is thought useful to present a general discussion concerning the universal character of the proposed model and to state that the involved constants and functional forms have not been simply choosen for data fitting.

The turbulence model (K-ϵ plus algebraic relations) has been previously used with success by us for situations different from Arnason's one, namely for the development of the

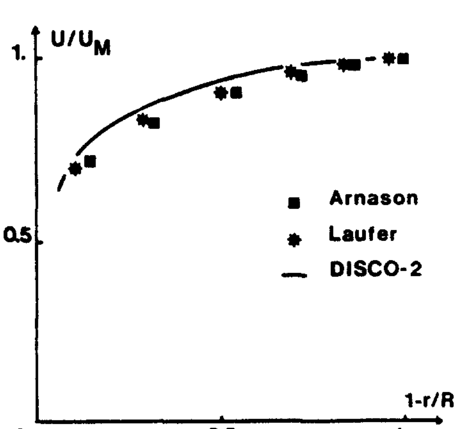

Figure 14. Mean velocity profiles in a turbulent pipe flow.

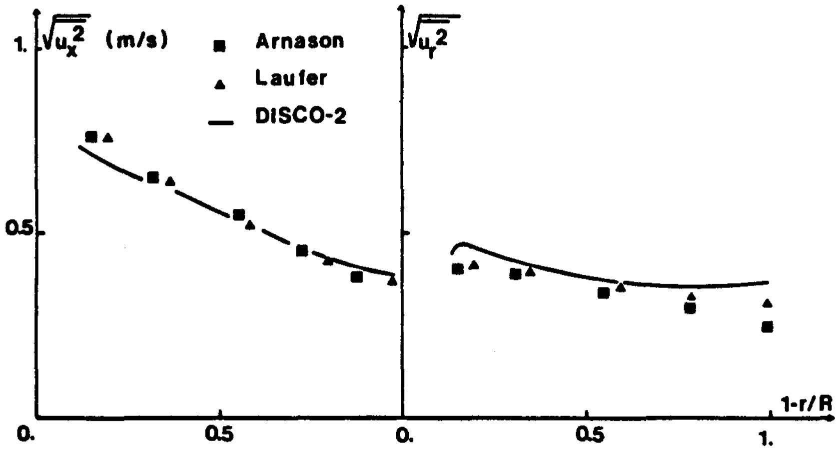

Figure 15. Longitudinal r.m.s.

Figure 16. Radial r.m.s.

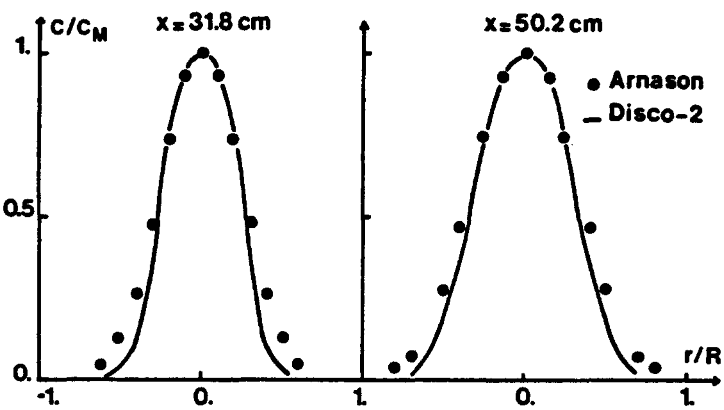

Figure 17. Comparison with Arnason's experiments.

Figure 18. Comparison with Arnason's experiments.

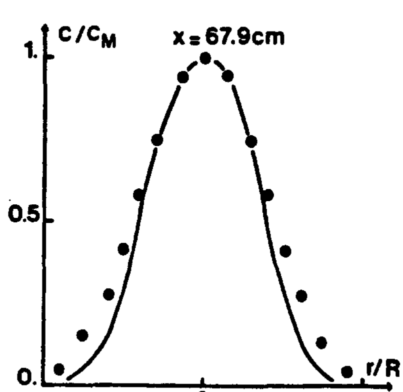

Figure 19. Comparison with Arnason's experiments.

pipe flow, the round free jet, (Berlemont & Gouesbet 1981), the channel flow and the confined mixing layer (unpublished). This model consequently owns some kind of universal character. Nevertheless, it is clear that we do not reach a full universality. People are trying to develop more sophisticated models and improvements of such models would result in improvements of our approach and extension of its range of applications.

Concerning the dispersion model, we have to discuss the Frenkiel form, the estimate of the macroscale τ_L and the correction for the CTE. The choosen Frenkiel form is not arbitrary. It is consistant with some experiments (Calabrese & Middleman 1979) and is a simple extension of the $\exp(-t/\tau_L)$ form, which is the only one discussed by Hinze (1959) in its celebrated book on turbulence. Furthermore, the approach in DISCO-1 was different to built the fluid Lagrangian correlation function $R_{fL}(\tau)$. The comparison between DISCO-1 and DISCO-2 results show that there is a fair insensitivity to the approach chosen to model $R_{fL}(\tau)$. The estimate for τ_L including the constant 0.2 has been used in previous works, specially for scalar diffusion (Dupont *et al.* 1984), before we heard of Arnason's results. The correction for the CTE relies on a previous analysis by Csanady (1963), Yudine (1959), and on a dimensional approach. The constant C_β was taken equal to 0.85 for data fitting but the guessed value $C_\beta \sim 1$ would have lead to similar results. Note that the value 0.85 is correct for both Snyder & Lumley (1971), and also Wells (1982) experiments. More generally, the fact that a good agreement between theory and experiments is obtained with the same approach for three different cases shows that our particle model also owns some kind of universal character. Again, as for the turbulence model, full universality is neither expected nor claimed. Any theoretical progress in predicting the function $R_{fL}(\tau)$, estimating τ_L or the correction for the CTE would lead to improvements. Furthermore, some assumptions are explicitly stated and relaxing these assumptions would also lead to improvements and extension of the range of applications. One example is that we do not know what is to be done when the diameter of the particles becomes larger than the Kolmogoroff scale. This is a challenging question for future work. We must also state that the experiments we have chosen have not been simply retained because they would be the only ones to compare favourably with our model. Actually, other experiments are discussed in the literature. Unfortunately, they generally cannot be used because our assumptions are not met (too big particles, larger than the Kolmogoroff scale, or volume fraction too important, for instance), or people do not measure all we need and the operating conditions are not well enough precised, or also, very often, the turbulence field is not properly enough designed and characterized. We are indeed faced to a very difficult problem for which experiments are too scarce. Future experiments are planned to increase the number of available data and to permit an interplay between theory and material reality. Waiting for these experiments, the present state of the art is certainly a good compromise between physical requirements and numerical simplicity.

12. CONCLUSION

Prediction of particle dispersion in turbulent flows has been presented. Turbulence fields are predicted with the aid of a $(K\text{-}\epsilon)$ model supplemented with algebraic relations deduced from a second-order closure scheme for the Reynolds tensor. The modelling of the discrete particle behavior is based on the nondiscrete dispersive approach leading to a transport equation for the mean number of particles per unit of volume which involves a dispersion tensor. A two-parameter family of Lagrangian correlation functions has been assumed to evaluate the dispersion tensor and the crossing trajectory effects are introduced into the code. Predictions have been compared with experimental results and the agreement is satisfactory. It must finally be stressed that, the agreement being satisfactory for three different experiments, the possible involved constants are expected to own some kind of universal character.

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